

13.29 Spin-orbit coupling is a magnetic interaction between spin and orbital magnetic moments. When the angular momenta are parallel, as in (a), the magnetic moments are aligned unfavourably; when they are opposed, as in (b), the interaction is favourable. This magnetic coupling is the cause of the splitting of a configuration into levels.

circulating current, and possesses a magnetic moment that arises from its orbital momentum. The interaction of the spin and orbital magnetic moments is called spin-orbit coupling. The strength of the coupling, and its effect on the energy levels of the atom, depends on the relative orientations of the spin and orbital magnetic moments, and therefore on the relative orientations of the two angular momenta (Fig. 13.29).

(a) The total angular momentum

One way of expressing the dependence of the spin-orbit interaction on the relative orientation of the spin and orbital momenta is to say that it depends on the total angular momentum of the electron, the vector sum of its spin and orbital momenta. Thus, when the spin and orbital angular momenta are nearly parallel, the total angular momentum is high; when the two angular momenta are opposed, the total angular momentum is low.

The total angular momentum of an electron is described by the quantum numbers  $j$  and  $m_j$ , with  $j = l + \frac{1}{2}$  (when the two angular momenta are in the same direction) or  $j = l - \frac{1}{2}$  (when they are opposed, Fig. 13.30). The different values of  $j$  that can arise for a given value of  $l$  label levels of a term. For  $l = 0$ , the only permitted value is  $j = \frac{1}{2}$  (the total angular momentum is the same as the spin angular momentum because there is no other source of angular momentum in the atom). When  $l = 1$ ,  $j$  may be either  $\frac{3}{2}$  (the spin and orbital angular momenta are in the same sense) or  $\frac{1}{2}$  (the spin and orbital angular momenta are in opposite senses).

Example 13.4 Identifying the levels of a configuration

Identify the levels that may arise from the configurations (a)  $d^1$ , (b)  $s^1$ .

Method In each case, identify the value of  $l$  and then the possible values of  $j$ . For these one-electron systems, the total angular momentum is the sum or difference of the orbital and spin momenta.

Answer (a) For a  $d$  electron,  $l = 2$  and there are two levels in the configuration, one with  $j = 2 + \frac{1}{2} = \frac{5}{2}$  and the other with  $j = 2 - \frac{1}{2} = \frac{3}{2}$ . (b) For an  $s$  electron  $l = 0$ , so only one level is possible, and  $j = \frac{1}{2}$ .

Self-test 13.7 Identify the levels of the configurations (a)  $p^1$  and (b)  $f^1$ .

[(a)  $\frac{3}{2}, \frac{1}{2}$ ; (b)  $\frac{7}{2}, \frac{5}{2}$ ]

The dependence of the spin-orbit interaction on the value of  $j$  is expressed in terms of the spin-orbit coupling constant,  $A$  (which is typically expressed as a wavenumber). A quantum mechanical calculation leads to the result that the energies of the levels with quantum numbers  $s, l$ , and  $j$  are given by

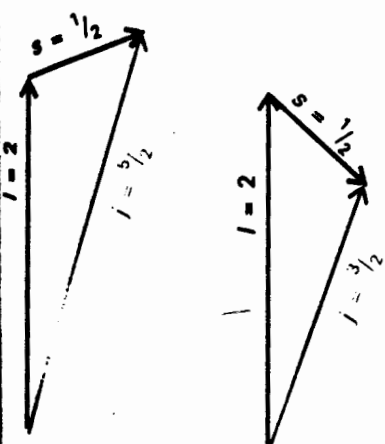
$$E_{l,s,j} = \frac{1}{2}hcA\{j(j+1) - l(l+1) - s(s+1)\} \tag{41}$$

Justification 13.10

The energy of a magnetic moment  $\mu$  in a magnetic field  $B$  is equal to their scalar product  $-\mu \cdot B$ . If the magnetic field arises from the orbital angular momentum of the electron, it is proportional to  $l$ ; if the magnetic moment  $\mu$  is that of the electron spin, then it is proportional to  $s$ . It then follows that the energy of interaction is proportional to the scalar product  $s \cdot l$ :

$$\text{energy of interaction} = -\mu \cdot B \propto s \cdot l$$

13.30 The coupling of the spin and orbital angular momenta of a  $d$  electron ( $l = 2$ ) gives two possible values of  $j$  depending on the relative orientations of the spin and orbital angular momenta of the electron.



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